ECL 4340

POWER SYSTEMS

LECTURE 2
REACTIVE COMPENSATION, POWER FACTOR CORRECTION, THREE-PHASE CIRCUITS

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ANNOUNCEMENT

- Please read Chapters 1 and 2
- HW 1; Project 1:– due Wednesday 9/1, in Canvas
  - in-class quiz, randomly administered
  - For Project, you need to use the PowerWorld Software. You can download the software and cases at the link below; get version 19 (August 6, 2018) http://www.powerworld.com/gloveroberbyesarma.asp

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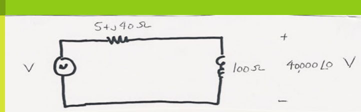
EXAMPLE



First solve basic circuit:

$$I = \frac{40000\angle 0^\circ\ V}{100\angle 0^\circ\ \Omega} = 400\angle 0^\circ \text{ Amps}$$

$$V = 40000\angle 0^\circ + (5 + j40)\ 400\angle 0^\circ$$
$$= 42000 + j16000 = 44.9\angle 20.8^\circ \text{ kV}$$

$$S = VI^* = 44.9\text{k}\angle 20.8^\circ \times 400\angle 0^\circ$$
$$= 17.98\angle 20.8^\circ \text{ MVA} \quad = 16.8 + j6.4 \text{ MVA}$$

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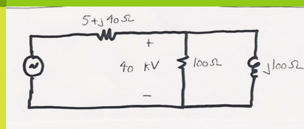
Example



Now add additional reactive power load & resolve:

$$Z_{Load} = 70.7\angle 45^\circ \quad pf = 0.7 \text{ lagging}$$
$$I = 564\angle -45^\circ \text{ Amps}$$
$$V = 59.7\angle 13.6^\circ \text{ kV}$$
$$S = 33.7\angle 58.6^\circ \text{ MVA} \quad = 17.6 + j28.8 \text{ MVA}$$

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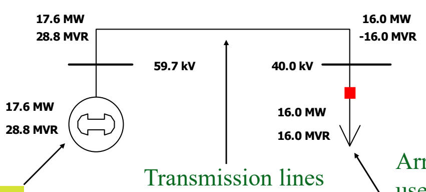
Power System Notation

Power system components are usually shown as "one-line diagrams." Previous circuit redrawn



Generators are shown as circles

Transmission lines are shown as a single line

Arrows are used to show loads

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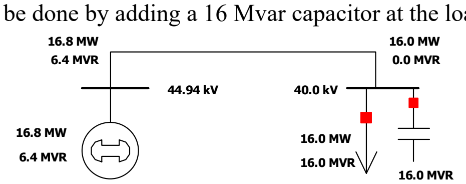
Reactive Compensation

Key idea of reactive compensation is to supply reactive power locally. In the previous example this can be done by adding a 16 Mvar capacitor at the load.



Compensated circuit is identical to the first example with just real power load.

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Reactive Compensation, cont'd

- Reactive compensation decreased the line flow from 564 Amps to 400 Amps. Advantages:
  - Lines losses, which are equal to $I^2 R$ decrease
  - Lower current allows utility to use small wires, or alternatively, supply more load over the same wires
  - Voltage drop on the line is less
- Reactive compensation is used extensively by utilities
- Capacitors can be used to "correct" a load's power factor to an arbitrary value.

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Power Factor Correction Example

Assume we have 100 kVA load with pf=0.8 lagging, and would like to correct the *pf* to 0.95 lagging

$$S = 80 + j60 \text{ kVA} \qquad \phi = \cos^{-1} 0.8 = 36.9^\circ$$

$$pf \text{ of } 0.95 \text{ requires } \phi_{\text{desired}} = \cos^{-1} 0.95 = 18.2^\circ$$

$$S_{new} = 80 + j(60 - Q_{cap})$$

$$\frac{60 - Q_{cap}}{80} = \tan 18.2^\circ \Rightarrow 60 - Q_{cap} = 26.3 \text{ kvar}$$

$$Q_{cap} = 33.7 \text{ kvar}$$

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Distribution System Capacitors

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Balanced 3 Phase (ϕ) Systems

- A balanced 3 phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
- Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial

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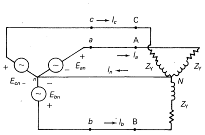
Balanced 3 Phase (ϕ) Systems



A three-phase (3ϕ) circuit is a circuit with three voltage (or current) sources:

For balanced 3ϕ circuits, 3 voltages are: <u>equal</u> in magnitude, but shifted by 120° in phase angle:

EX.

$E_{an} = 100\angle 0°$

$E_{bn} = 100\angle -120° = 100\angle +240°$

$E_{cn} = 100\angle -240° = 100\angle +120°$

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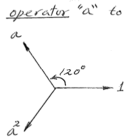
Balanced 3 Phase (ϕ) Systems

an operator "a" to rotate (shift) the phasors:



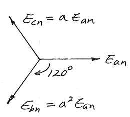
$a = 1\angle 120° = e^{j120°}$

$= -\frac{1}{2} + j\frac{\sqrt{3}}{2}$

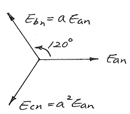
Note: $a^2 = a \cdot a = e^{j240°} = 1\angle 240°$

$a^3 = a \cdot a^2 = 1\angle 0°$

Thus, the three-phase voltage can be defined as:



Positive sequence



Negative sequence

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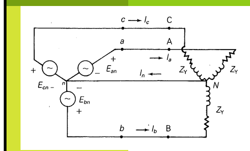
Balanced 3 Phase (ϕ) Systems



Returning to the 3ϕ circuit, the current through each line can be computed as:

$$I_a = \frac{E_{an}}{Z_Y} = \frac{|E_{an}|\angle 0^\circ}{|Z_Y|\angle\theta} = \frac{|E_{an}|}{|Z_Y|}\angle -\theta = |I_a|\angle -\theta = I_a$$

$$I_b = \frac{E_{bn}}{Z_Y} = \frac{|E_{an}|\angle -120^\circ}{|Z_Y|\angle\theta} = |I_a|\angle -\theta - 120^\circ = I_a\angle -120^\circ$$

$$I_c = \frac{E_{cn}}{Z_Y} = \frac{|E_{an}|\angle -240^\circ}{|Z_Y|\angle\theta} = |I_a|\angle -\theta - 240^\circ = I_a\angle -240^\circ$$

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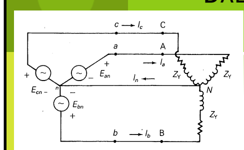
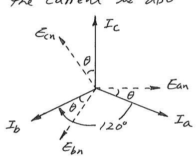
Balanced 3 Phase (ϕ) Systems



Thus, for balanced source and the load (Y connected), the current is also balanced!



Note that the current in the neutral line is

$$I_n = I_a + I_b + I_c = 0$$

for "balanced" 3ϕ circuit.

Since the current is zero the potential between "n" & "N" is also zero. Therefore, we can "remove" the neutral line, and yet the circuit remains the same in operation.

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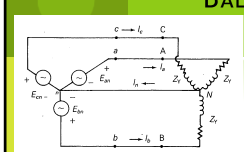
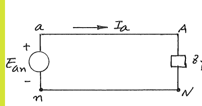
Balanced 3 Phase (ϕ) Systems



Since we now know that the current is balanced, we only need to compute phase-"a" current and then rotate to find phase-"b" and phase-"c" currents. Thus, we can simply use the single-phase equivalent:



Caution: Often the neutral line has some impedance (could be large for ground). But you should not include the neutral line impedance in the 1ϕ-equivalent circuit.

Why?

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Advantages of 3ϕ Power

- Can transmit more power for the same amount of wire (twice as much as single phase)
- Torque produced by 3ϕ machines is constant
- Three-phase machines use less material for the same power rating
- Three-phase machines start more easily than single phase machines

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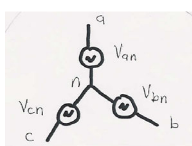
Three Phase - Wye Connection

- There are two ways to connect 3ϕ systems
  - Wye (Y)
  - Delta (Δ)

Wye Connection Voltages

$$V_{an} = |V|\angle\alpha^\circ$$
$$V_{bn} = |V|\angle\alpha^\circ - 120^\circ$$
$$V_{cn} = |V|\angle\alpha^\circ + 120^\circ$$



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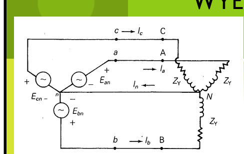
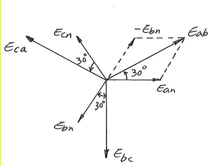
Wye Connection Line Voltages



Line-to-line voltages: Normally, we prefer to use the voltages between lines rather than the voltages between lines and the neutral. (Easier to access).



Voltage of line "a" with respect to line "b" is defined by:

$$E_{ab} = E_{an} - E_{bn}$$
$$= E_{an} - a^2 E_{an}$$
$$= (1 - a^2) E_{an}$$
$$= (1 - (-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2})) E_{an}$$
$$= (\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2}) E_{an}$$
$$= (\sqrt{3}\angle 30^\circ) E_{an}$$

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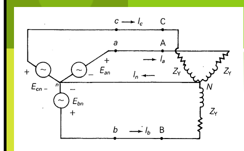
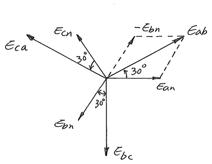
Wye Connection Line Voltages





Voltage of line "a" with respect to line "b" is defined by:

$$
\begin{aligned}
E_{ab} &= E_{an} - E_{bn} \\
&= E_{an} - a^2 E_{an} \\
&= (1 - a^2)\, E_{an} \\
&= \left(1 - \left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right)\right) E_{an} \\
&= \left(\tfrac{3}{2} + j\tfrac{\sqrt{3}}{2}\right) E_{an} \\
&= \left(\sqrt{3}\,\angle 30^\circ\right) E_{an}
\end{aligned}
$$

Similarly,

$$E_{bc} = E_{bn} - E_{cn} = (\sqrt{3}\,\angle 30^\circ)\, E_{bn}$$

$$E_{ca} = E_{cn} - E_{an} = (\sqrt{3}\,\angle 30^\circ)\, E_{cn}$$

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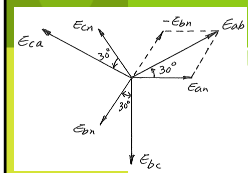
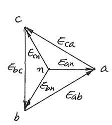
Wye Connection Line Voltages



Thus, we conclude that the line-to-line voltages, or simply line voltages, are also balanced, and are $\sqrt{3}$ times the line-to-neutral voltages and lead by 30°.

The phasor diagram above can be rearranged to have the following voltage triangle:



Note the "clockwise" rotation of the vertices a b c indicates the positive sequence.

Note: $E_{ab} + E_{bc} + E_{ca} \equiv 0$

Sum of line voltages is always equal to zero (even for unbalanced case!)

However,

$$E_a + E_b + E_c = 0$$

only for the balanced voltages.

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Wye Connection, cont'd

- Define voltage-across/current-through a device to be *phase voltage/current*
- Define voltage-across/current-through a line to be *line voltage/current*

$$
\begin{aligned}
V_{Line} &= \sqrt{3}\, V_{Phase}\, 1\angle 30^\circ = \sqrt{3}\, V_{Phase}\, e^{j\pi/6} \\
I_{Line} &= I_{Phase} \\
S_{3\phi} &= 3\, V_{Phase}\, I^*_{Phase}
\end{aligned}
$$

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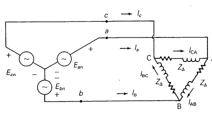
Delta Connection

Δ - connected loads:



Consider a 3ϕ Y-connected source feeding a balanced Δ-connected (delta-connected) load, i.e., three identical impedance Z_Δ is connected in Δ.

Then, the currents in each impedance (Δ-load current) are:

$$I_{AB} = \frac{E_{ab}}{Z_\Delta}$$

$$I_{BC} = \frac{E_{bc}}{Z_\Delta} = \frac{E_{ab}}{Z_\Delta}\angle -120^\circ$$

$$I_{CA} = \frac{E_{ca}}{Z_\Delta} = \frac{E_{ab}}{Z_\Delta}\angle -240^\circ$$

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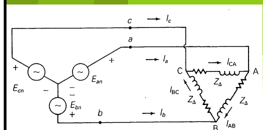
Delta Connection

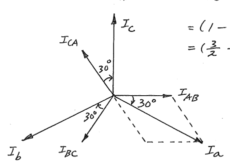


Since 3ϕ source voltages are balanced, Δ-currents are also balanced. Besides the Δ-currents, we also need to know the line currents (I_a, I_b, I_c), which can be determined by the KCL. For example, at node A,

$$I_a = I_{AB} - I_{CA} = I_{AB} - aI_{AB} = (1-a)I_{AB}$$

$$= (1 - (-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}))I_{AB}$$

$$= (\tfrac{3}{2} - j\tfrac{\sqrt{3}}{2})I_{AB} = (\sqrt{3}\angle -30^\circ)I_{AB}$$



Similarly,

$$I_b = I_{BC} - I_{AB} = (\sqrt{3}\angle -30^\circ)I_{BC}$$

$$I_c = I_{CA} - I_{BC} = (\sqrt{3}\angle -30^\circ)I_{CA}$$

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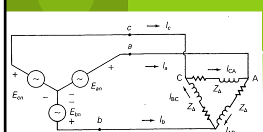
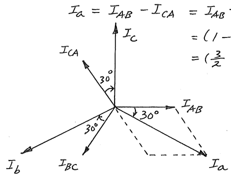
Delta Connection



$$I_a = I_{AB} - I_{CA} = I_{AB} - aI_{AB} = (1-a)I_{AB}$$

$$= (1 - (-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}))I_{AB}$$

$$= (\tfrac{3}{2} - j\tfrac{\sqrt{3}}{2})I_{AB} = (\sqrt{3}\angle -30^\circ)I_{AB}$$



Similarly,

$$I_b = I_{BC} - I_{AB} = (\sqrt{3}\angle -30^\circ)I_{BC}$$

$$I_c = I_{CA} - I_{BC} = (\sqrt{3}\angle -30^\circ)I_{CA}$$

Thus, we conclude that the line currents are also balanced, and are $\sqrt{3}$ times the Δ-currents (or phase currents) and lags by 30°.

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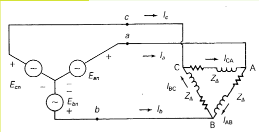
Delta Connection



NOTE: $I_a + I_b + I_c \equiv 0$ (Sum of line voltages are always equal to zero, even for unbalanced case!! Why?)

However,
$I_{AB} + I_{BC} + I_{CA} = 0$ only for balanced case. Why?

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Delta Connection

- Define voltage-across/current-through a device to be *phase voltage/current*
- Define voltage-across/current-through a line to be *line voltage/current*

$$
\begin{aligned}
I_{Line} &= \sqrt{3}\, I_{Phase}\, 1\angle -30^\circ = \sqrt{3}\, I_{Phase}\, e^{-j\pi/6} \\
V_{Line} &= V_{Phase} \\
S_{3\phi} &= 3\, V_{Phase}\, I^*_{Phase}
\end{aligned}
$$

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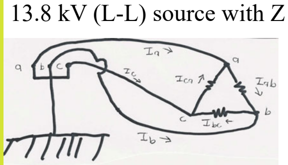
Three Phase Example

Assume a Δ-connected load is supplied from a 3ϕ 13.8 kV (L-L) source with Z = 100∠20° Ω



$$
\begin{aligned}
V_{ab} &= 13.8\angle 0^\circ\ kV \\
V_{bc} &= 13.8\angle -120^\circ\ kV \\
V_{ca} &= 13.8\angle 120^\circ\ kV
\end{aligned}
$$

$$I_{ab} = \frac{13.8\angle 0^\circ\ kV}{100\angle 20^\circ\ \Omega} = 138\angle -20^\circ\ amps$$

$$I_{bc} = 138\angle -140^\circ\ amps \qquad I_{ca} = 138\angle 100^\circ\ amps$$

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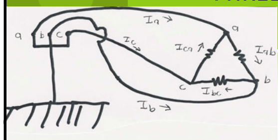
Three Phase Example, cont'd



$$
\begin{aligned}
I_a &= I_{ab} - I_{ca} = 138\angle -20° - 138\angle 100° \\
&= 239\angle -50° \text{ amps} \\
I_b &= 239\angle -170° \text{ amps} \quad I_c = 239\angle 70° \text{ amps} \\
S &= 3 \times V_{ab} I_{ab}^{*} = 3 \times 13.8\angle 0° \text{kV} \times 138\angle 20° \text{ amps} \\
&= 5.7\angle 20° \text{ MVA} \\
&= 5.37 + j1.95 \text{ MVA} \\
\text{pf} &= \cos 20° = 0.94 \text{ lagging}
\end{aligned}
$$

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Delta-Wye Transformation

To simplify analysis of balanced 3ϕ systems:

1) Δ-connected loads can be replaced by Y-connected loads with $Z_Y = \frac{1}{3} Z_\Delta$

2) Δ-connected sources can be replaced by Y-connected sources with $V_{phase} = \frac{V_{Line}}{\sqrt{3}\angle 30°}$

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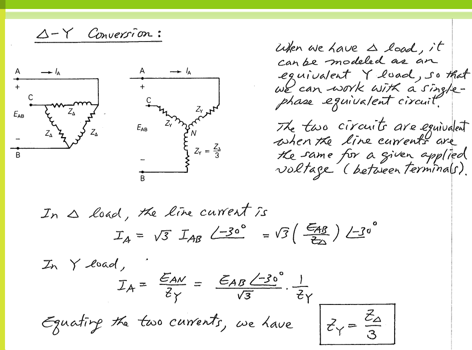
Delta-Wye Transformation Proof



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Three-Phase Transmission Line

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